

## Surface-wave propagation along the boundary of a plasma with nonlocal anisotropic electric properties

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We have developed an effective description of surface-wave propagation along the boundary between vacuum and a dense plasma or a low-density and high-density plasma in the case when the high-density plasma can be characterized by a nonlocal asymmetric conductivity tensor. This last condition leads to a different dispersion relation from the well-known one for surface waves. The asymmetric conductivity significantly affects the phase velocity and attenuation of the surface wave and makes the conditions for its excitation simpler. The effects of energy dissipation in the dense plasma on the evanescent and surface waves are taken into consideration. It is shown that damping of the surface wave in the propagation direction is low, if the surface impedance is small ( $|\xi| \ll 1$ ). We also discuss the possibilities of excitation surface waves in different geometries.

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### I. INTRODUCTION

The development of the laser facilities [1] generating high-intensity ( $I > I_a$ , where  $I_a = cE_a^2/8\pi = 3.4 \times 10^{16}$  W/cm<sup>2</sup> and  $E_a = e/r_b^2 = 5.1 \times 10^9$  V/cm is the atomic field) ultrashort pulses ( $< 1$  psec) allows the creation of laboratory plasmas with solid-state density and unusual properties (i.e., properties different from those of a classical laser-produced plasma characterized by nanosecond time scales). In the presence of the strong electromagnetic field ( $E > E_a$ ) atoms in a solid target are ionized first due to multiphoton ionization in the tunnel limit, and second by electron-impact ionization in a time shorter than the period of the laser radiation [2]. The important feature of plasmas generated on the femtosecond time scale is that there is not enough time to convert the electron energy into kinetic energy of directed motion of the ions and hence insignificant hydrodynamic motion occurs during the pulse. At the very least, a very steep density gradient [ $(d \ln n_i / dx)^{-1} \ll \lambda_0$ , where  $n_i$  is the ion density and  $\lambda_0$  is the wave length of the incident radiation] will develop over an interaction time of about a hundred femtoseconds [3]. Another important property of a dense plasma created in the skin layer of the radiation is that at electron temperatures higher than several keV the plasma becomes collisionless. In this case the electron mean free path  $l_{ei}$  substantially exceeds the penetration depth of the electromagnetic field into the plasma,  $l_s$ , e.g., skin depth, so  $l_{ei} \gg l_s$ . In this case the relationship between the current density and the associated electric field is nonlocal and, in general, is represented by an asymmetric conductivity tensor. The spatial distribution of the evanescent electromagnetic field within the plasma is then no longer exponential, as in the absence of spatial dispersion, and has to be determined by solving the self-consistent problem of the field penetration and absorption along with the kinetic equation for the electron distribution function [4].

We have shown qualitatively in a previous paper [5] that the distinct boundary between the low-density plasma (or vacuum) and high-density plasma, which has the nonlocal properties, can support surface waves. The properties of the dense plasma influence the dispersion relation via the surface impedance. Generally, the surface impedance is proportionality coefficient between electric and magnetic field at the interface where the electromagnetic wave is incident. For the media having anisotropic electric properties the surface impedance appears to be the two-dimensional (2D) tensor [6,8]. The surface impedance in general form obeys the Kramers-Kronig relations similar to those for the dielectric permeability. For the anisotropic plasma with nonlocal properties to be considered later in this paper the surface impedance (2D tensor) depends on the anisotropic conductivity via the anisotropic electron distribution function and can be found after solution of Maxwell equations using proper boundary conditions.

In this paper we present and analyze the dispersion relations for surface waves quantitatively in different target geometries for isotropic and asymmetric electron distribution functions with different dissipation mechanisms taken into account. It is shown that damping of the surface wave may be small in the case when the surface impedance is small. The change of the degree of asymmetry of the conductivity (or the electron distribution function) leads to a change of the ratio of the real to imaginary parts of the impedance and consequently to a change in the dispersion relation. We also discuss the possibility of exciting a surface wave propagating along the boundary of a dense plasma and an inhomogeneous plasma with a steep density gradient by an obliquely incident  $p$ -polarized heating beam which also creates the plasma.

### II. FORMULATION OF THE PROBLEM

Let us consider a set of simple geometries where the plane  $z = 0$  separates the half space  $z > 0$  occupied by a

plasma characterized by a constant ion density, cold immobile ions, and hot electrons. At  $z < 0$  we consider three cases: (a) vacuum; (b) a homogeneous slab of low-density plasma; and (c) a slab of plasma with a very steep density gradient (the density changes abruptly from 0 to the plasma density at  $z > 0$ ). The final geometry corresponds to a plasma created by the interaction of an intense subpicosecond laser pulse with a solid target [2–4]. Our aim is to find solutions which represent surface waves, propagating along the boundary  $z = 0$ . It is assumed that the electromagnetic field associated with the surface wave has the following form:

$$\mathbf{E}(0, E_y, E_z), \mathbf{H}(H, 0, 0) \sim \exp(-i\omega t +iky),$$

where  $k$  is the component of the wave vector in the direction of propagation of the wave and  $\omega$  is the wave frequency. We also assume that the amplitude of the field associated with the surface wave is small, causing only a perturbation of the background distribution function. Because the electromagnetic field is evanescent in the dense plasma, the frequency of a surface wave is small compared with the plasma frequency  $\omega < \omega_{pe} = (4\pi e^2 N_e / m)^{1/2}$ , where  $e$ ,  $N_e$ , and  $m$  are, respectively, the electron charge, density, and mass. The spatial distribution of the electromagnetic field is then described by the set of Maxwell's equations

$$\nabla \times \mathbf{H} = \frac{4\pi}{c} \mathbf{j} + \frac{1}{c} \frac{\partial \mathbf{E}}{\partial t}, \quad \nabla \times \mathbf{E} = -\frac{1}{c} \frac{\partial \mathbf{H}}{\partial t}, \quad (1)$$

where  $\mathbf{j}$  is the current density and  $c$  is the speed of light in vacuum. Equivalently the field is described by the following set of coupled wave equations:

$$\frac{\partial^2 E_y}{\partial z^2} + k_0^2 E_y - ik \frac{\partial E_z}{\partial z} + \frac{4\pi}{c} ik_0 j_y = 0, \quad (2)$$

$$(k_0^2 - k^2) E_z - ik \frac{\partial E_y}{\partial z} + \frac{4\pi}{c} ik_0 j_z = 0, \quad (3)$$

where  $k_0 = \omega/c$ . In the high-density plasma ( $z > 0$ ) the current density  $\mathbf{j}$  is nonlocally related to the electric field  $\mathbf{E}$  via the conductivity tensor  $\sigma$ ,

$$i_\alpha = \int_0^\infty \sigma_{\alpha\beta}(z, z') E_\beta(z') dz', \quad (4)$$

$(\alpha, \beta) = (z, y).$

In a rare plasma ( $\omega \gg \omega_{pe1}$ ), where  $\text{Re}\epsilon \gg \text{Im}\epsilon$ , (2) and (3) reduce to the usual wave equation for dielectrics by introducing  $\epsilon_1 = 1 + i4\pi\sigma_1/\omega = 1 - \omega_{pe1}^2/\omega^2$ . According to Ref. [6] the spatial distribution of the electromagnetic field within the dense plasma can be related to the surface impedance via inverse Fourier transformation

$$E_y(z) = \frac{1}{2\pi} \int_{-\infty}^\infty \xi(q) H_0 e^{iqz} dq, \quad (5)$$

where  $H_0 = H(z = +0)$  and the surface impedance is

$$\xi = \frac{1}{2\pi} \int_{-\infty}^\infty \xi(q) dq. \quad (6)$$

Let us look for a solution where a  $p$ -polarized electromagnetic wave propagates along the boundary  $z = 0$  between the low-density homogeneous and high-density plasma and is evanescent both to the left and right of the boundary. In the rare plasma the electromagnetic wave is characterized by well-known solutions  $E, H \sim \exp\{k_1 z - i\omega_0 t +iky\}$  and

$$E_y = \frac{ik_1}{k_0 \epsilon_1} H. \quad (7)$$

Here  $k_1^2 = k^2 - \epsilon_1 k_0^2 > 0$ ;  $0 < \epsilon_1 < 1$ . Requiring continuity of  $E_y, H$  at  $z = 0$  one obtains from (5)–(7) the following dispersion relation for the surface waves

$$\frac{ik_1}{k_0 \epsilon_1} = \xi. \quad (8)$$

In the case of a medium with the local properties at  $z > 0$ , which is characterized by dielectric permeability  $\epsilon_2$  and  $k_2^2 = k^2 - k^2 \epsilon_2 (|\mathbf{E}|, |\mathbf{H}| \sim \exp\{-k_2 z\})$  the surface impedance in (8) can be replaced by

$$\xi \rightarrow -\frac{ik_2}{k_0 \epsilon_2} \quad (9)$$

and one arrives at the usual dispersion relation for surface waves [7,8]

$$\frac{k_1 \epsilon_2}{\epsilon_1 k_2} = -1. \quad (10)$$

The relation (10) describes the surface-wave solutions only for the case  $\epsilon_2 < 0$ . In what follows we will obtain the surface impedances for different electron distribution functions in the dense plasma, and analyze the corresponding dispersion relations for surface waves. Note that the dispersion relation (8) is also valid for the case of vacuum at  $z < 0$  ( $\epsilon_1 = 1$ ).

### III. THE SURFACE IMPEDANCE IN A PLASMA WITH NONLOCAL ELECTRIC PROPERTIES

To Fourier transform the wave equations (2) and (3) one has to continue the field quantities into the region  $z < 0$ . Assuming the electrons are specularly reflected from the boundary at  $z = 0$ , the field quantities are continued into the region  $z > 0$  in accordance with the usual recipe [6]

$$E_y(-z) = E_y(z), \quad H(-z) = -H(z), \quad (11)$$

$$E_z(-z) = -E_z(z).$$

The component  $E_y$  is continuous across the boundary  $z = 0$ , while  $E_z$  and  $dE_y/dz$  at the left ( $z < 0$ ) and the right side ( $z > 0$ ) are related via the boundary condition [6]

$$\left[ ikE_z - \frac{\partial E_y}{\partial z} \right]_{z=+0} - \left[ ikE_z - \frac{\partial E_y}{\partial z} \right]_{z=-0} = 2ik_0 H_0, \quad (12)$$

$$H_0 = H(z=0).$$

Let us define the Fourier transform  $f_q$  of a function  $F(z)$  in a usual manner,

$$\phi_q = \int_{-\infty}^{\infty} F(z) e^{iqz} dz. \quad (13)$$

Now one can Fourier transform the wave equations (2) and (3) over the whole space  $-\infty < z < \infty$  making use of (4) and (11)–(13). Such a transform then results in the following set of the coupled equations:

$$E_{yq} = 2ik_0 H_0 \left\{ \left[ i \frac{4\pi k_0}{c} \sigma_{yy} - (q^2 - k^2) \right] - \frac{[kq + i(4\pi k_0/c)\sigma_{zy}][kq + i(4\pi k_0/c)\sigma_{yz}]}{(k_0^2 - k^2) + i(4\pi k_0/c)\sigma_{zz}} \right\}^{-1}, \quad (15)$$

$$E_{yq} \equiv \xi(q) H_0.$$

The spatial distribution of the electromagnetic field within the plasma then can be calculated by (5).

Let us now calculate the surface impedance for the different cases assuming that the background electron distribution function is known and not affected by the surface wave. One can calculate the conductivity by using the routine perturbation method [6]. The electron distribution function is represented as the sum of a slowly varying bulk part (on the time scale of the pulse duration) and small rapidly varying part (changing on the time scale of the period of the laser wave).

After linearization of the kinetic equation one can relate the rapidly varying part to the slowly varying one and apply this relation for the calculation of the conductivity. In the calculations of the skin effect in metals at low intensities the electron distribution function in the skin layer is assumed to be Maxwellian during the whole interaction time [6,8]. At high intensities ( $> 10^{16}$  W/cm<sup>2</sup>) and for subpicosecond pulses the slowly varying part of the electron distribution function is strongly affected by incident electromagnetic field and is transient, non-Maxwellian, and asymmetric [4,9]. In general, Maxwell's equations and the kinetic equation must be solved self-consistently. In this case the penetration depth of the field (the skin length) depends on the form of the electron distribution function via an integral relation [9].

Using the routine perturbation procedure one can obtain the Fourier transform for the electric current in the plasma for the general case of an arbitrary (relativistic) electron distribution function

$$j_\alpha(q) = \sigma_{\beta\alpha}(q) E_\beta(q) = E_y \sigma_{y\alpha} + E_z \sigma_{z\alpha}, \quad (16)$$

where  $\alpha, \beta \rightarrow y, z$  and

$$\xi_s = \frac{1}{2\pi} \int_{-\infty}^{\infty} \xi(q) dq = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{2ik_0 |q| dq l_s^3}{i - |q|(q^2 - k^2) l_s^3 - k^2 q^4 l_s^6 / [(k_0^2 - k^2)|q| l_s^3 + i]}. \quad (20)$$

$$\left[ (k_0^2 - k^2) + \frac{4i\pi k_0}{c} \sigma_{zz} \right] E_{zq} + \left[ kq + i \frac{4\pi k_0}{c} \sigma_{zy} \right] E_{yq} = 0, \quad (14)$$

$$2ikH_0 = E_{yq} \left[ i \frac{4\pi k_0}{c} \sigma_{yy} - (q^2 - k^2) \right] + \left[ i \frac{4\pi k_0}{c} \sigma_{yz} + kq \right] E_{zq}.$$

Solving (14) for  $E_{yq}$  one obtains

$$\sigma_{y\alpha} = e^2 \int_0^\infty \frac{u_\alpha}{|u_z|} \frac{2aid^3 p}{(a^2 - q^2)} \times \left\{ \frac{\partial f}{\partial p_y} - \frac{a}{|u_z|} \left[ u_z \frac{\partial f}{\partial p_y} - u_y \frac{\partial f}{\partial p_z} \right] \right\},$$

$$\sigma_{z\alpha} = e^2 \int_0^\infty \frac{u_\alpha}{|u_z|} \frac{2aid^3 p}{(a^2 - q^2)} \times \left\{ \frac{\partial f}{\partial p_z} + \frac{k}{k_0 c} \left[ u_z \frac{\partial f}{\partial p_y} - u_y \frac{\partial f}{\partial p_z} \right] \right\}, \quad (17)$$

$$a = \frac{\omega_0 - ku_y}{|u_z|}.$$

Here  $f(\mathbf{p}, t)$  is the slowly varying bulk part of the electron distribution function, and  $e$ ,  $\mathbf{u}$ , and  $\mathbf{p}$  are the electron charge, velocity, and momentum, respectively.

#### A. Isotropic case

For the case of an isotropic nonrelativistic distribution function  $f(p^2)$  (but one which is not necessarily Maxwellian) one can obtain the diagonal terms of the symmetric conductivity tensor by integrating (17) over the angles between  $\mathbf{p}$  and the  $z$  axis,

$$\sigma_{yy} = \sigma_{zz} = \frac{\sigma_0}{|q|}, \quad \sigma_0 = 2\pi e^2 \int_0^\infty p \frac{\partial f}{\partial p} dp_y dp_x. \quad (18)$$

Note that  $\sigma_0 \sim \omega_{pe}^2 / \langle u \rangle$ , where  $\omega_{pe}$  is the electron plasma frequency and  $\langle u \rangle$  is the average electron velocity. Let us introduce the skin length as

$$l_s = \left[ \frac{4\pi k_0 \sigma_0}{c} \right]^{-1/3} \sim \frac{c}{\bar{\omega}_{pe}} \left[ \frac{\langle u \rangle}{c} \frac{\bar{\omega}_{pe}}{\bar{\omega}} \right]^{1/3}. \quad (19)$$

Now making use of (15), (5), (18), and (19) one can obtain the surface impedance for this case in the form

We are interested in surface waves in the electromagnetic limit, when  $k > k_0$ . On the other hand, the skin length is much less than laser wavelength, i.e.,  $k_0 l_s \ll 1$ . The integrand of (20) peaks near  $q^{-1} \sim l_s$ . Taking into account these considerations one can reduce (20) to the form

$$\xi_s = (1 + k^2 l_s^2 e^{i\pi/3}) k_0 l_s e^{-i\pi/3} \frac{2}{\pi} \int_0^\infty \frac{|\chi| d\chi}{1 + |\chi|^2}. \quad (21)$$

The integral in (21) has tabulated value

$$\frac{1}{\pi} \int_0^\infty \frac{|\chi| d\chi}{1 + |\chi|^2} = \frac{1}{3\pi} \frac{\Gamma(\frac{1}{3})\Gamma(\frac{2}{3})}{\Gamma(1)} = 0.385.$$

The final form of the surface impedance is

$$\xi_s = 0.77 k_0 l_s (e^{-i\pi/3} + k^2 l_s^2) = \text{Re}\xi_s + i \text{Im}\xi_s. \quad (22)$$

### B. Anisotropic case ( $\sigma_{zy} \neq \sigma_{yz} \neq 0$ )

The anisotropy of the electron distribution function depends on the intensity of the incident wave. At relatively low intensities ( $I > 10^{16}$  W/cm<sup>2</sup>) the electrons gain energy when accelerated in the direction of the propagation of the electric field of the incident wave (in parallel to the surface). The energy of these electrons slightly exceeds the energy of those moving in  $z$  direction ( $T_\perp > T_z$ ,  $T_\perp/T_z = 1.1-1.2$ ) [4]. For the intensities in excess of the relativistic one ( $I > I_r \sim 10^{18}$  W/cm<sup>2</sup>), the ponderomotive force becomes of major importance and drives electrons in the direction of the incident wave. The anisotropy of the electron distribution function changes for this case in such a way that  $T_z > T_\perp$  [10,11].

Analysis of the denominator in Eq. (15) shows that in this case the last term containing the anisotropic conductivity is dominated by the product of the off-diagonal terms of the conductivity tensor ( $4\pi k_0/c$ )<sup>2</sup>  $\sigma_{zy}\sigma_{yz}$ . This term is proportional to  $\omega_{pe}^2 l_s^2/c^2$  while the other terms are proportional to  $k^2 l_s^2$  and  $kl_s$ , respectively (note that  $\omega_{pe} \gg \omega$ ). In the case of the anomalous skin effect collisionless case) the electron time of flight through the skin layer is less than the wave period and the collision time, i.e.,  $\omega/qu_z \ll 1$ . Taking into account this condition and  $\omega_{pe} \ll \omega$  one can expand the surface impedance of Eq. (15) into a power series of  $\omega/qu_z$  in the same manner as was done in [4]. This results in a relation for the surface impedance similar to (22) (see [4]),

$$\xi_{as} = k_0 l_s \frac{2}{\pi} \int_0^\infty \frac{udu}{1 + u(u^2 - d)} \equiv k_0 l_s \Phi(d), \quad (23)$$

where  $d = (\omega_{pe} l_s/c)^2 \Delta$  and  $\Delta$  is the anisotropy parameter for the relativistic electron distribution function,

$$\Delta = \frac{1}{n_e} \int dp \left\{ \left[ 1 + \frac{p_x^2 + p_y^2/2}{m^2 c^2} \right] \gamma^{-3} f_0 + \frac{p_x^2}{2p_z \gamma} \frac{\partial f}{\partial p_z} \right\}, \quad (24)$$

where  $p_\perp^2 = p_x^2 + p_y^2$  and  $\gamma = (1 + p^2/m^2 c^2)^{1/2}$  is the relativistic factor. For the case of a two-temperature nonrelativistic electron distribution function

$$f_e(u) = \frac{n_e m^{3/2}}{(2\pi T_z)^{1/2} 2\pi T_\perp} \exp \left[ -\frac{mu_z^2}{2T_z} - m \frac{u_x^2 + u_y^2}{2T_\perp} \right], \quad (25)$$

Eq. (25) reduces to

$$\Delta = \frac{T_\perp}{T_z} - 1. \quad (26)$$

The real and imaginary parts of (24) were calculated numerically in [4]. In the case of  $d=0$  ( $\Delta=0$ , the isotropic function), Eqs. (21) and (23) coincide to within an accuracy of  $k^2 l_s^2$ . The asymmetry can change the ratio of the real to the imaginary parts of the impedance by up to ten times in comparison with the isotropic case. This occurs due to the change in the relative role of two absorption mechanisms: noncollisional absorption (like Landau damping) and capacitor heating. Note that the influence of the asymmetry on the surface impedance may be significant even in the case when the asymmetry parameter  $\Delta$  is very small. Formally this is connected with the fact that the anisotropy parameter  $\Delta$  is multiplied by the factor  $(\omega_{pe} l_s/c)^2$  which may be larger than unity in the case of the anomalous skin effect due to  $\omega_{pe} \gg \omega$ . If  $\Delta > 1$  ( $T_\perp > T_z$ ) noncollisional absorption dominates and  $\text{Re}\Phi > |\text{Im}\Phi|$  ( $\text{Re}\xi_{as} > |\text{Im}\xi_{as}|$ ). In the case  $\Delta < 0$  ( $T_z > T_\perp$ ) capacitor heating prevails and  $|\text{Im}\xi_{as}| > \text{Re}\xi_{as}$ .

### IV. THE DISPERSION RELATIONS FOR THE SURFACE WAVES

Now one can write the dispersion relation (8) for the surface waves in an explicit form introducing the complex surface impedance by the relation

$$\xi = |\xi| e^{i\phi} \equiv \text{Re}\xi + i \text{Im}\xi \equiv R + iI,$$

$$\cos\phi = \frac{R}{(R^2 + I^2)^{1/2}}.$$

Equation (8) reduces to

$$\frac{k^2}{k_0^2} = \epsilon_1 - \epsilon_1^2 |\xi|^2 e^{i2\phi}. \quad (27)$$

Taking into account that the surface impedance is small  $|\xi| \sim k_0 l_s \ll 1$  one can solve Eq. (27) for the  $\mathbf{k}$  vector to obtain

$$\begin{aligned} \text{Re}k &= k_0 \epsilon_1^{1/2} (1 - \epsilon_1 |\xi|^2 \cos 2\phi)^{1/2}, \\ \text{Im}k &= \text{Re}k \frac{\epsilon_1 |\xi|^2 |\cos 2\phi|}{2\sqrt{2}}, \end{aligned} \quad (28)$$

where  $\cos 2\phi = (R^2 - I^2)/(R^2 + I^2)$ .

For the case of an isotropic distribution function (in the electromagnetic limit for the surface wave  $k > k_0$ ), one obtains  $|\xi_s| \sim 0.77 k_0 l_s$ ,  $\phi = \pi/3$ , and (28) reduces to

$$\begin{aligned} \text{Re}k &= k_0 \epsilon_1^{1/2} \left\{ 1 + a \left[ \frac{\bar{\omega}}{\bar{\omega}_{pe}} \right]^{4/3} \left[ \frac{\langle u \rangle}{c} \right]^{2/3} \right\}^{1/2}, \\ \text{Im}k &= \text{Re}k \frac{\epsilon_1 |\xi|^2}{4\sqrt{2}}, \end{aligned} \quad (29)$$

where  $\mathbf{a}$  is a numerical coefficient of the order of unity. The real part of (29) coincides with the qualitative estimates from [5]. It follows from (29) that attenuation of the surface wave is low if the surface impedance is small. For the case of an anisotropic electron distribution function one can deduce from (23) that

$$\xi_{\text{as}} = (k_0 l_s)(\text{Re}\Phi + i \text{Im}\Phi). \quad (30)$$

After insertion of (30) into (28) it is easy to calculate the phase velocity of the surface wave on the plasma boundary

$$V_{\text{phase}} = \frac{\bar{\omega}}{\text{Re}k} = \frac{c}{\epsilon_1^{1/2}} \left\{ 1 - \epsilon_1 |\xi_{\text{as}}|^2 \frac{(R_{\text{as}}^2 - I_{\text{as}}^2)}{(R_{\text{as}}^2 + I_{\text{as}}^2)} \right\}^{-1/2}. \quad (31)$$

This equation describes the main difference between the anisotropic and isotropic cases. In the isotropic case  $|I| > R$  ( $R > 0$ ,  $I < 0$ ), and the phase velocity is always lower than speed of light in vacuum. For the case of an anisotropic function at  $\Delta > 1$ , as it follows from [4],  $R \geq |I|$ . Thus if  $R$  approaches  $|I|$ , the phase velocity of the surface wave approaches the speed of the light in vacuum even in the case of vacuum-plasma boundary ( $\epsilon_1 = 1$ ). It means that unlike the classical case for excitation of a surface wave, it is possible to excite such a wave on the boundary of any medium with an anisotropic conductivity. Note that when  $R$  approaches  $|I|$  the attenuation of the surface wave ( $\text{Im}k$ ) goes to zero. At the given wave number  $k$  it is easy to deduce from (31) the decrement of the surface wave damping

$$\gamma = \text{Im}k \left[ \frac{d\omega}{dk} \right] \sim \text{Im}k \left[ \frac{c}{\epsilon_1^{1/2}} \right]. \quad (32)$$

The decrement is low if the surface impedance is small,  $|\xi| \ll 1$ .

The form of the electron distribution function can also change the skin-length dependence of the plasma and electromagnetic wave parameters. For the anisotropic case (equivalent to a stationary, bi-Maxwellian electron distribution function) the skin length depends on the ratio of the transverse to longitudinal temperatures [4].

$$l_{s,\text{as}} = \frac{c}{\omega_p} \left[ \frac{u_z \omega_p}{c \omega} \right]^{1/3} \left[ \frac{2 T_z}{\pi T_1} \right]^{1/3}.$$

In this case the skin length changes only by a small numerical coefficient  $[(2/\pi)(T_z/T_1)]^{1/3}$ . When a powerful ultrashort laser pulse interacts with a solid target, a plasma with a transient, non-Maxwellian electron distribution function is created. The skin length in such a plasma is expressed as [9]

$$l_{\text{tr}} \sim \frac{c}{\omega_{pe}} \left[ \frac{u_{\text{os}}}{c} \right]^{1/3} (\omega t)^{1/6}, \quad (33)$$

where  $u_{\text{os}} = eE_0/m_e\omega$  is the electron oscillation velocity in the electric field of amplitude  $\mathbf{E}$ . The surface impedance for this case reads

$$\xi \cong k l_{\text{tr}} = \left[ \frac{\omega}{\omega_{pe}} \right]^{2/3} \left[ \frac{eE_0}{\omega_{pe} m_e c} \right]^{1/3} (\omega t)^{1/6}. \quad (34)$$

Finally, for all cases of electron distribution functions which have been considered (isotropic, anisotropic, transient, non-Maxwellian), the main frequency dependence of the surface impedance remains the same,  $\sim (\omega/\omega_{pe})^{2/3}$ . Only a weak correction  $\sim (\omega t)^{1/6}$  appears in the case of a time-dependent distribution function.

## V. DISCUSSION

Let us now compare the dispersion relations for a medium with nonlocal and asymmetric electric properties to the known relations for different media with local properties. For the simplest case of two dielectrics with  $0 < \epsilon_1 < 1$  at  $z < 0$  and  $\epsilon_2 < 0$ ,  $|\epsilon_2| > \epsilon_1$  at  $z > 0$  from (10) one can easily obtain the linear dispersion relation. In this case  $k$  vector of the surface wave has only a real part [8],

$$k^2 = \frac{\omega^2}{c^2} \frac{|\epsilon_2| \epsilon_1}{|\epsilon_2| - \epsilon_1}. \quad (35)$$

Let us suppose that a metal is located at  $z > 0$  with  $\text{Im}\epsilon_2 = \epsilon_2'' = 4\pi\sigma/\omega$  and  $\text{Re}\epsilon_2 = \epsilon_2' \ll \epsilon_2''$ . It is easy to deduce from (10)

$$\text{Re}k \cong k_0 \epsilon_1^{1/2} \left\{ 1 + \frac{3}{4} \frac{\epsilon_1^2}{(\epsilon_2'')^2} \right\}, \quad (36)$$

$$\text{Im}k \cong \text{Re}k \frac{\epsilon_1}{\sqrt{2}\epsilon_2''}.$$

For highly conducting metals, if  $v_{\text{eff}} > \omega_p > \omega$ ,  $\text{Re}\sigma \approx \omega_p^2/4\pi v_{\text{eff}}$  ( $v_{\text{eff}}$  is the effective collisional frequency), and  $\epsilon_2'' \approx \omega_p^2/\omega v_{\text{eff}}$  the mode of the interaction falls into the frame of the normal skin effect [8]. In this case

$$(\text{Re}k_2)^{-1} \sim l_s = \frac{\epsilon}{(4\pi\sigma\omega)^{1/2}}. \quad (37)$$

One can reduce (36) to

$$\text{Re}k = \frac{\omega}{c} \epsilon_1^{1/2} \left[ 1 + \frac{3}{4} \left[ \frac{\omega}{\omega_p} \right]^2 \left[ \frac{v_{\text{eff}}}{\omega_p} \right]^2 \right], \quad (38)$$

$$\text{Im}k = \text{Re}k \frac{\epsilon_1 \omega v_{\text{eff}}}{\sqrt{2}\omega_p^2}.$$

It is important to note that the formal introduction of the surface impedance for this case,

$$\xi = |\xi| e^{-i3\pi/4} = \frac{\omega l_s}{c} e^{-i3\pi/4},$$

into Eq. (8) leads to an incorrect form of the dispersion relations. The physical reason for this is transparent: Eq. (8) describes correctly only the case of the medium with nonlocal electric properties.

For the last example of a medium with local properties let us consider at  $z > 0$  the dense plasma ( $\omega_p \gg \omega$ ) with low collisional losses ( $\omega \gg v_{\text{eff}}$ ). The dielectric permeability for this case reads [12]

$$\epsilon_2 = 1 - \frac{\omega_p^2}{\omega^2} + i \frac{\omega_p^2 \gamma}{\omega^3} \equiv \epsilon_2' + i \epsilon_2'', \quad (39)$$

where  $\epsilon_2' < 0$ ,  $|\epsilon_2''| > \epsilon_2'$ , and  $|\epsilon_2''| \gg \epsilon_1$ .

Introducing (39) into (10) and solving for the  $k$  vector one can get

$$\begin{aligned} \text{Re}k &= k_0 \epsilon_1^{1/2} \left[ 1 + \frac{\epsilon_1}{|\epsilon_2'|} \right]^{1/2} = k_0 \epsilon_1^{1/2} \left[ 1 + \frac{\epsilon_1 \omega^2}{\omega_p^2} \right], \\ \text{Im}k &= \text{Re}k 2 \epsilon_1^2 \frac{\omega^2}{\omega_p^2} \frac{\nu_{\text{eff}}}{\omega}. \end{aligned} \quad (40)$$

A comparison of all these cases shows the following. In the case of a nondissipative medium ( $\text{Im}\epsilon_2=0$ ) the dispersion relation is linear and depends only on the values of the dielectric permeabilities of both media. Introducing dissipation (in the case of the metal:  $\text{Im}\epsilon_2 \neq 0$ ;  $\text{Re}\epsilon_2=0$ ) leads to a nonlinear dispersion relation. The introduction of the large negative real part of the permeability along with low collisional dissipation in the plasma does not change the main frequency dependence of the nonlinear part: it remains the same as in the case of a metal,  $\sim \omega^2/\omega_p^2$ . If one considers a surface wave propagating along the boundary between a low- and high-density plasma with nonlocal isotropic properties, the nonlinear part of the dispersion relation changes only slightly,  $\sim (\omega/\omega_p)^{4/3}$ , in comparison with a plasma which has local properties. The transition to media having nonlocal *anisotropic* properties corresponds to the introduction of a new parameter: the degree of anisotropy. This parameter appears in the dispersion relation as the ratio of the real to the imaginary part of the impedance. The real part of the impedance is positive in accordance with the Kramers-Kronig relations [8]. The imaginary part is negative and for the case of the isotropic anomalous skin effect  $|\text{Im}\xi|/\text{Re}\xi = \sqrt{3}$  [6]. For an anisotropic plasma this ratio decreases as the anisotropy parameter increases,  $\Delta = (T_1/T_z) - 1 > 0$ . As follows from calculations [4]  $|\text{Im}\xi|/\text{Re}\xi = 1$  at  $d \approx 1$ , e.g.,  $T_1 \leq 2T_z$ .

Thus on the boundary of such a plasma the phase velocity of the surface wave may approach the speed of light in vacuum. Consequently it is possible, in principle, to excite the surface wave even on the boundary of such a plasma and vacuum.

To understand the physical reason for this phenomenon one has to remember that not only nonlocality characterizes this kind of the plasma. The anomalous skin effect arises when two conditions are fulfilled. First, the electron mean free path substantially exceeds the field penetration depth, e.g.,  $l_{ei} \gg l_s$ . The second condition dictates that the distance the electron penetrates during a wave period is larger than the skin length, e.g.,  $u/\omega \gg l_s$ . Thus the first condition is responsible for spatial dispersion of the conductivity, while the second relates to frequency dispersion. In fact, the anisotropy changes not only the relative role of the different absorption mechanisms but strongly affects the dispersion properties of the plasma. It is worth comparing the similarity of the results obtained here with the case of a surface wave on the boundary of a nonlinear medium. For such

a medium the changes in refractive index are due to dependence of the permeability on the electric field intensity. Because these changes are largest near the surface, they create a channel supporting a surface wave. For the case of a plasma having nonlocal parameters, the change of the dispersive properties with distance inside a plasma is related to the change of the asymmetry parameter of the electron distribution function. This asymmetry has a maximum near the surface and decreases inside the plasma thus causing the wave to have maximum phase velocity near the surface. Note also that the noncollisional absorption mechanism, which is responsible for the asymmetry in a nonlocal medium, is itself nonlinear (the absorption term depends on the field intensity).

It is out of the scope of this paper to discuss the methods for creating such a highly anisotropic plasma. The interaction of an ultrashort laser pulse with a solid target at the relativistic ( $I\lambda^2 > 10^{18} \text{ W}\mu\text{m}^2/\text{cm}^2$ ) intensities may be one of the possible ways. We should note that previous arguments are applicable also to the case of a medium with a highly anisotropic conductivity.

It is well known [13] that TE surface waves can be excited on the boundary of a homogeneous dielectric and a semi-infinite periodic layered medium. On the interface of two homogeneous dielectrics it is possible to excite only TM surface modes (as in the present paper). The question arises: is it possible to excite TE modes on the boundary of the highly anisotropic metal?

## VI. CONCLUSION

An effective description has been formulated of surface waves propagating along a high-density plasma-vacuum boundary or a high-density-plasma-low-density-plasma boundary in the case when the high-density plasma can be characterized by a nonlocal asymmetric conductivity tensor. Such a plasma with a steplike (or very steep) density gradient can be created by the interaction of a short ( $< 1$  ps) high-intensity ( $I\lambda^2 > 10^{18} \text{ W}\mu\text{m}^2/\text{cm}^2$ ) laser pulse with a solid target. Assuming that the plasma electron distribution function is known and only slightly affected by the surface wave, we have calculated the surface impedances for isotropic and anisotropic plasmas. We have found and analyzed the dispersion relations for the surface waves for these cases. Attenuation of the waves has been taken into account.

It is shown that the frequency dependence in the dispersion relation remains the same for the different forms of the electron distribution function. On the other hand, the introduction of the asymmetry parameter, which is related to the ratio of the real to the imaginary parts of the surface impedance, leads to an increase in the phase velocity of the surface wave. Physically this indicates a change in the relative role of the different absorption mechanisms in the plasma and relates to the role of dispersion on the surface wave. In an isotropic medium the noncollisional damping (phase breaking) is dominant while in an anisotropic plasma the role of capacitor heating, connected with the work done by the  $z$  component of the electric field on the electrons, increases. Thus it is possible to excite a surface wave even on a

vacuum–anisotropic-plasma interface if the proper asymmetry parameter can be obtained. The same arguments are also applicable to the case of a medium with a highly anisotropic conductivity tensor. We plan to consider this problem elsewhere.

The next step for this work is to consider the full nonlinear self-consistent problem in which the surface wave is excited by the heating beam itself. It should be noted that this problem was partially considered in the present paper. It has been shown in Sec. III that a surface wave in the electromagnetic limit ( $k > k_0$ ) only weakly affects the skin depth and the attenuation properties. We proposed previously [5] the use of layered targets (consisting of a low-density foam and a metal layer) which, after being ionized by an intense ultrashort heating laser beam, allow the excitation of surface waves by an obliquely incident  $p$ -polarized electromagnetic wave. It is worth not-

ing that a similar structure is created during the interaction of an ultrashort intense laser pulse with the solid. It was shown both theoretically [3] and experimentally [14] that a thin inhomogeneous plasma layer with very steep density gradient of the order  $\ll \lambda_0$  ( $\lambda_0$  is the laser wave length) is created on the surface of a metal target (during the interaction time of hundreds of femtoseconds). This layer can play the same role as a low-density plasma layer in supporting surface waves [15] on the boundary of the plasma with solid density.

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